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BALANCING WITH AN OFFSET MASS CENTER

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In Specifying Unbalance and the Location of Tolerance Planes, a rotor with mass center equidistant from the two bearing planes is considered. In the examples offered, the static unbalance is referred to as force or single plane unbalance and is equally divided between the right and left tolerance planes which are the same as the bearing planes. This is a good illustration of balancing fundamentals; however, bearing reactions can be significantly different for a rotor with an offset mass center. An axial offset exists when the mass center is not located in the plane midway between the bearings. This paper outlines three cases that illustrate the effects of an offset mass center.

The reader should be familiar with the basic concepts of “static” and “dynamic” unbalance. Each is related to the mass properties of the spinning body and varies with the location and orientation of the spin axis with respect to the body. They are entirely independent of each other. The bearing reactions due to static unbalance vary with the location of the mass center, cases 1 and 2 deal with this effect. For case 1 the mass center is located between the bearings and in case 2 the mass center is located outside the bearings. When a correction is made in a plane that does not pass through the mass center, a dynamic unbalance is created. The bearing reactions generated by a dynamic unbalance vary only with the distance between bearings, case 3 deals with this effect.

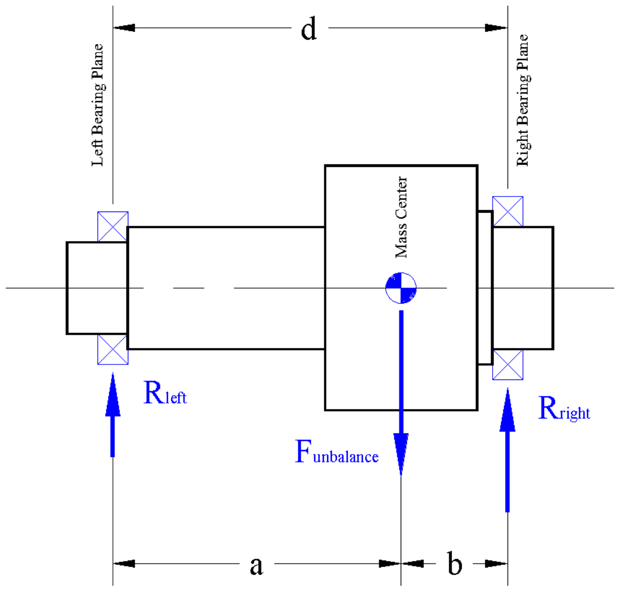
A static or “force” unbalance exists when the mass center of the body does not lie exactly on the spin axis. The magnitude of static unbalance can be used to calculate the force generated at a given spin speed. Static unbalance can be corrected by adding or removing weight on the line of action of the unbalance force. The effect of this weight is to move the mass center of the body closer to or onto the spin axis. Bearing reaction forces due to static unbalance vary greatly according to the location of the mass center in relation to the bearings. Furthermore, if the correction plane does not contain the mass center, a dynamic unbalance will be created in the correction process.

A dynamic unbalance exists when the principal inertia axes of the body are not aligned with the spin axis. This is often referred to as “moment” or “couple” unbalance and can be interpreted as 2 static unbalances some distance apart. The magnitude of dynamic unbalance can be used to calculate the moment or couple created at a known spin speed. Reaction forces at the bearings are inversely proportional to the distance between bearings. These reactions can be significant in cases where the bearings are close together or where the static correction plane is a large distance away from the mass center plane.

Variables and units for all cases are defined at the end of this paper. Most should be familiar. The constant, K, is used to generate the proper units in equations that combine weight, mass and speed. It has a different value and units depending on the system of units used – english or metric.

CASE 1

The mass center is located between the bearings and the correction plane passes through the mass center. A static unbalance exists which is the total unbalance of the part either before or after correction. There is no dynamic unbalance.



The unbalance force is

$$F_{unbalance} = U_{static} \cdot \omega^2 \cdot K.$$

From a simple static analysis, the reaction forces at the bearings are

$$R_{left} = ({}^{b}/_{d}) \cdot U_{static} \cdot \omega^2 \cdot K$$

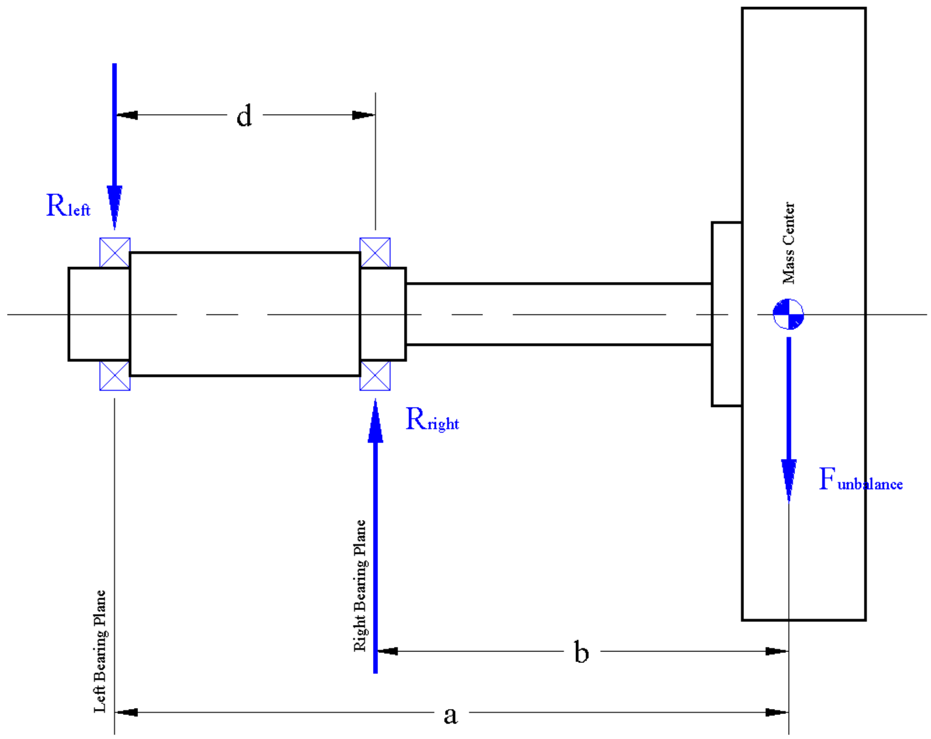
$$R_{right} = ({}^{a}/_{d}) \cdot U_{static} \cdot \omega^2 \cdot K.$$

For a = b, the left and right bearing reactions are each equal to half of the total unbalance force. In the extreme case, b = 0 and a = d, the right bearing reacts the entire unbalance force and the left bearing reaction is zero.

The left and right reaction forces are in phase with each other for this case (except when a or b = 0).

CASE 2

The mass center is located outside the bearings and the correction plane passes through the mass center. A static unbalance exists which is the total unbalance of the part either before or after correction. There is no dynamic unbalance.



The unbalance force is

$$F_{unbalance} = U_{static} \cdot \omega^2 \cdot K.$$

From a simple static analysis, the reaction forces at the bearings are

$$R_{left} = (^b/_d) \cdot U_{static} \cdot \omega^2 \cdot K$$

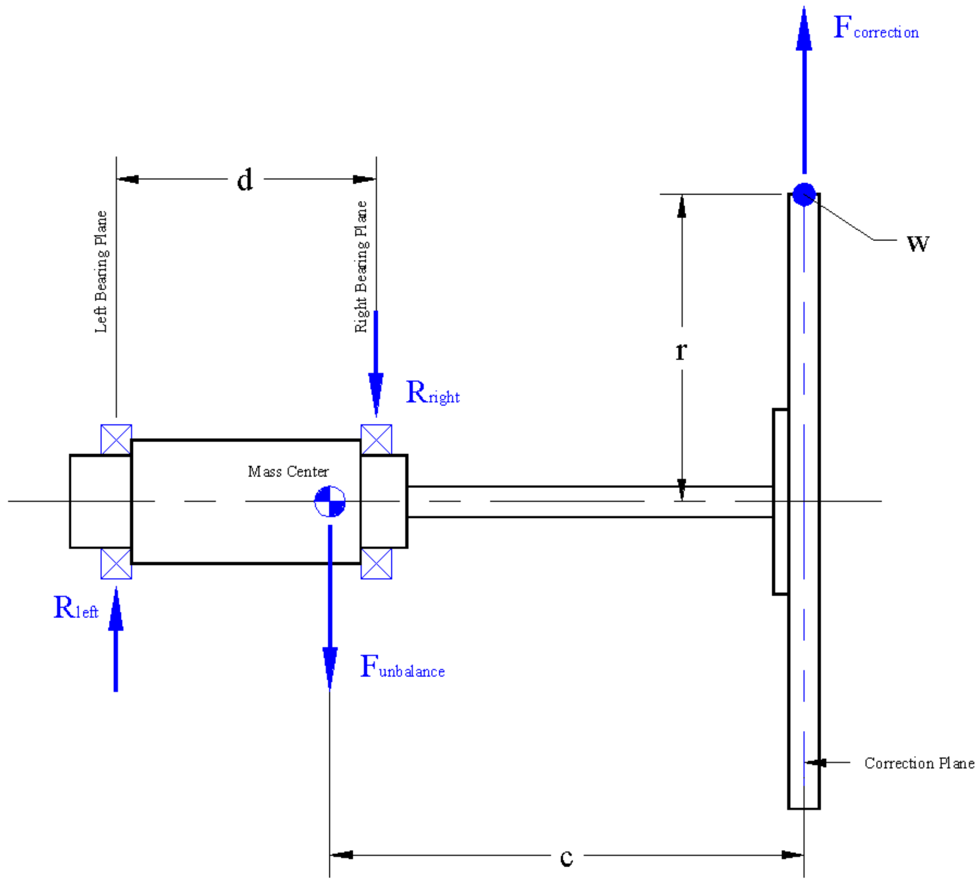
$$R_{right} = (^a/_d) \cdot U_{static} \cdot \omega^2 \cdot K.$$

When b is large compared to d, the bearing reactions can be very large. Possibly many times <u>greater</u> than the unbalance force itself. The reaction at the right bearing is always greater than the reaction at the left bearing.

The left and right reaction forces are 180° out of phase with each other for this case (b>0). Note that this is the only difference between cases 1 and 2. The force equations are identical, only the direction of the left bearing reaction is reversed.

CASE 3

The mass center is located a distance, c, from the correction plane. The location of the mass center with respect to the bearings is not required for this analysis. There is no dynamic unbalance before correction.



The part has an initial static unbalance, U_{static} , which creates an unbalance force

$$F_{unbalance} = U_{static} \cdot \omega^2 \cdot K.$$

Assume that a correction is made to the unbalanced rotor by placing a weight, w, at a radius, r, in the correction plane. The correction is exactly equal to the initial unbalance and is precisely placed so

$$w \cdot r = U_{static}$$

and

$$F_{correction} = F_{unbalance}.$$

While the correction reduces the static unbalance to zero, it creates a dynamic unbalance equal to the amount of the correction times the distance between the mass center and correction plane

$$U_{dynamic} = w \cdot r \cdot c.$$

This, together with any initial dynamic unbalance, forms a residual dynamic unbalance. The moment created by the residual unbalance is reacted by the bearings. These reactions are computed using a static analysis

$$R_{left} = R_{right} = ({}^{1}/_{d}) \cdot U_{dynamic} \cdot \omega^2 \cdot K$$

$$= ({}^{c}/_{d}) \cdot U_{static} \cdot \omega^2 \cdot K.$$

The bearing reactions have the same magnitude, but are 180° out of phase.

For $c = 0$, the mass center actually lies in the correction plane and there is no residual unbalance. This is the same as case 1 or 2.

For $0 < c < d$, the reaction forces are less than the forces which would have been created by the uncorrected static unbalance.

For $c > d$, the reaction forces are greater than those due to the uncorrected static unbalance.

It is not possible to eliminate the residual dynamic unbalance without adding a second correction plane. A two plane correction provides the ability to make a complete correction for both static and dynamic unbalance. The analysis outlined in these three cases can be used to evaluate plane locations.

In the event that the static correction is not perfect as assumed earlier, there will also be a residual static unbalance. The reactions due to the static unbalance can be determined as outlined in cases 1 and 2. These reactions will add to those from the residual dynamic unbalance. The phase of the unbalances, if known, should be considered to accurately predict magnitude of the total residual unbalance.

CONCLUSIONS

The location of the mass center with respect to the correction plane and with respect to the bearings is a key consideration when designing a part for balance. The spacing between the bearings themselves is also important. For single plane correction, the correction plane should be placed near the mass center. If this is not practical, a second plane may be necessary to adequately balance the rotor. Prudent choices made during design will minimize bearing reaction forces, which are one of the best means of evaluating the effects of rotor balance. They are, of course, a key consideration in bearing selection also.

This paper assumes that the shafts are rigid – stresses in the spinning shaft are small and deflections insignificant. For slender shafts or high stresses this may not be a good assumption. Shaft deflection can create a speed dependent balance characteristic that greatly complicates balancing. More thorough modeling and analysis is required for this situation.

VARIABLES AND UNITS

Variables and units for all cases are listed below. All of the equations can be evaluated using english or metric units. Be sure to use the units suggested below and the appropriate K value.

a,b,c,d and r are lengths in (inches) or (mm).

w is a weight in (oz) or (g).

ω is the speed of rotation in (radians/sec).

U_{static} is static unbalance in (oz in) or (g mm).

$U_{dynamic}$ is dynamic unbalance in (oz in^2) or (g mm^2).

$F_{unbalance}$ is a centrifugal force due to unbalance in (lbs) or (N).

$F_{correction}$ is a centrifugal force due to a correction weight in (lbs) or (N).

R_{left} and R_{right} are reaction forces in (lbs) or (N).

K is a constant used to generate the proper units in equations that combine weight, mass and speed quantities. Two values for the units recommended above are:

$$K_{english} = 1.619 \times 10^{-4} \text{ (lb sec}^2\text{)/(in oz)}$$

$$K_{metric} = 1.000 \times 10^{-6} \text{ (kg m)/(g mm)}$$

Check your units – it is easy to make mistakes.

Please address questions or comments on this paper to haidler@balancetechnology.com.